Kinetics of a migration-driven aggregation process with birth and death

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We propose an irreversible aggregation model driven by migration and birth-death processes with the symmetric migration rate kernel $K(k;j)=K'(k;j)=Ikj^{v}$, and the birth rate J_1k and death rate J_2k proportional to the aggregate's size k. Based on the mean-field theory, we investigate the evolution behavior of the system through developing the scaling theory. The total mass M_1 is reserved in the $J_1=J_2$ case and increases exponentially with time in the $J_1>J_2$ case. In these cases, the long-time asymptotic behavior of the aggregate size distribution $a_k(t)$ always obeys the scaling law for the $v\leqslant 2$ case. This model may provide a more natural description for diverse aggregation processes such as the evolution of the distribution of city population and individual wealth.

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I. INTRODUCTION

Aggregations are popular and important phenomena in natural science with abound kinetic evolution behavior through their complex mechanisms [1–5]. Much research effort has been devoted to the growth of aggregates and considerable understanding has been achieved on the evolution behavior of various aggregation processes over the past few decades [6–12]. The general mechanisms arising in diverse branches of physics are binary coalescence, annihilation, and so on. Recently, much attention has migrated to the aggregation phenomena in sociology and economy to investigate the kinetic behavior driven by some new mechanisms [13–16]. Ispolatov *et al.* introduced several asset exchange models for the evolution of the wealth distribution in the economical interaction population [17], and Leyvraz and Redner proposed a migration-driven aggregate growth model for the evolution of city populations [18]. In these models, irreversible growth of aggregates takes place through a migration mechanism, where there exists preferential migrations of monomers (or equivalently, units of assets or persons) from smaller aggregates to larger aggregates. The mechanism can be described by an irreversible reaction scheme, $A_k+A_l \xrightarrow{K(k;l)} A_{k-1}+A_{l+1}$ $(k\leqslant l)$, where A_k denotes an aggregate characterized only by its size k and $K(k;l)$ is the migration rate dependent on the sizes of the reactants. The solution to the rate equation exhibited that the kinetics of this process obeys a very different scaling law from that of the conventional aggregation process. In fact, the class of the migration-driven aggregation phenomena occurs in many branches of physics and social sciences [19].

More generally, the migration could also go from a larger aggregate to a smaller one as Leyvraz and Redner pointed out [18]. In our previous paper, we investigate the kinetics of the general migration-driven aggregation system, in which migration goes from the larger aggregates to the smaller as well as from the smaller to the larger [20]. The reaction rate at which a monomer emigrates from the aggregate A_k to the aggregate A_j is $K(k;j)\propto kj^{v}$ and that a monomer immigrates to the aggregate A_k from the aggregate A_j is $K'(k;j)\propto kj^{v}$. Here v is the migration rate kernel index, which may interpret the degree of "richness" in the population of a city. When the value of v increases, the city becomes much generous in emigration and much greedy in immigration. We find that for the $v\leqslant 2$ case, the evolution of the system always obeys a conventional scaling law.

In fact, we should realize that the birth and death play much more important roles in the evolution of the city population. Motivated by these new growth mechanisms, we investigate an aggregation process driven by migration and birth-death processes to model the city population evolution process much really. The immigration and emigration schemes for the aggregate A_k in our model are $A_k+A_j \xrightarrow{K(k;j)} A_{k+1}+A_j^{-1}$ and $A_k+A_l \xrightarrow{K'(k;l)} A_{k-1}+A_l^{+1}$, respectively. Here A_l^{+1} represents an aggregate with one temporary resident, and A_j^{-1} denotes an aggregate with one temporary empty resident. We assume that these two aggregates react immediately according to the scheme $A_l^{+1}+A_j^{-1} \xrightarrow{L(l;j)} A_l+A_j$ with the reaction rate $L(l;j)\to\infty$. Meanwhile, the birth and death schemes are $A_k \xrightarrow{J_1(k)} A_{k+1}$ and $A_k \xrightarrow{J_2(k)} A_{k-1}$, respectively. Thus our model may be believed to mimic some social and economical processes more naturally.

II. MODEL OF THE MIGRATION-DRIVEN AGGREGATION PROCESS WITH BIRTH AND DEATH

We assume that the system has spatial homogeneity, so that the fluctuations in the densities of the reactants are ignored and the aggregates are considered to be homogeneously distributed in the space throughout the processes. Thus, the theoretical approach to investigate the kinetics of the aggregation process can be based on the rate equation, which assumes that the reaction proceeds with a rate proportional to the reactant concentrations. Let $a_k(t)$ be the concentration of the aggregates A_k at time t. We generalize the rate equation of the migration-driven aggregation process given by Ref. [18] and write the corresponding rate equation for our system as follows:

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$$\frac{da_k}{dt}=\sum_{j=1}^{\infty}K(k+1;j)a_{k+1}a_j+\sum_{j=1}^{\infty}K'(k-1;j)a_{k-1}a_j$$
$$-\sum_{j=1}^{\infty}[K(k;j)+K'(k;j)]a_ka_j+J_1(k-1)a_{k-1}$$
$$-J_1(k)a_k+J_2(k+1)a_{k+1}-J_2(k)a_k, \tag{1}$$

where we impose the boundary condition $a_0(t)=0$.

For simplicity, we consider a model with a special migration rate kernel. The rate of the aggregate A_k gaining one monomer from the aggregate A_j or losing one monomer to the aggregate A_j through migration is directly proportional to its size k and j^{υ}, i.e., $K(k;j)=K'(k;j)=Ikj^{\upsilon}$ (I is a constant). This kernel may embody the conscious activity of the aggregates in the exchange process. The rate of the aggregate A_k propagating or extinguishing one monomer is directly proportional to its size k, i.e., $J_1(k)=J_1k$ and $J_2(k)=J_2k$ (J_1 and J_2 are two constants). To modify the demographic growth, which typically gives a population increasing exponentially with time [18], we consider the $J_1\geqslant J_2$ cases. With the above mentioned reaction rate definition, Eq. (1) reduces to

$$\frac{da_k}{dt}=IM_{\upsilon}(t)[(k+1)a_{k+1}-2ka_k+(k-1)a_{k-1}]$$
$$+J_1[(k-1)a_{k-1}-ka_k]+J_2[(k+1)a_{k+1}-ka_k], \tag{2}$$

where $M_{\upsilon}(t)=\Sigma_{j=1}^{\infty}j^{\upsilon}a_j(t)$ are the moments of the aggregate size distribution $a_k(t)$.

We assume that there only exist the monomer aggregates at $t=0$ and the concentration is equal to A_0. Then the initial conditions are $a_k(0)=A_0\delta_{k1}$.

Equation (2) can be solved with the help of ansatz [9]

$$a_k(t)=A(t)[a(t)]^{k-1}. \tag{3}$$

Substituting Eq. (3) into Eq. (2), we can transform the rate equation (2) into the following differential equations:

$$\frac{da}{dt}=[IM_{\upsilon}(1-a)+J_1-J_2a](1-a),$$

$$\frac{dA}{dt}=-[2IM_{\upsilon}(1-a)+(J_1+J_2-2J_2a)]A, \tag{4}$$

with the corresponding initial conditions

$$a=0,\quad A=A_0\quad \text{at } t=0. \tag{5}$$

From Eq. (4), we can find the basic and important feature of $a(t)$, i.e., $a(t)$ grows monotonously from its initial value 0 to the steady state value 1 for the $J_1\geqslant J_2$ case. To investigate the kinetic behavior of the system thoroughly, we study Eq. (4) for the cases of $J_1=J_2$ and of $J_1>J_2$.

III. KINETICS OF THE SYSTEM IN THE $J_1=J_2$ CASE

For the $J_1=J_2$ case, Eq. (4) reduces to

$$\frac{da}{dt}=(IM_{\upsilon}+J_1)(1-a)^2,\quad \frac{dA}{dt}=-2(IM_{\upsilon}+J_1)(1-a)A. \tag{6}$$

From this set of equations, we can directly find out the relation between $A(t)$ and $a(t)$ as follows:

$$A(t)=A_0[1-a(t)]^2. \tag{7}$$

This reveals the total mass of system conserved:

$$M_1=\sum_{j=1}^{\infty}ja_j=A\sum_{j=1}^{\infty}ja^{j-1}=\frac{A}{(1-a)^2}\equiv A_0. \tag{8}$$

To make a general analysis, we can use the property that the total mass of the system is conserved to rewrite Eq. (2) as follows:

$$\frac{da_k}{dt}=\left[IM_{\upsilon}(t)+\frac{J_1}{A_0}M_1\right][(k+1)a_{k+1}-2ka_k$$
$$+(k-1)a_{k-1}]. \tag{9}$$

From this equation, it immediately follows that the $J_1=J_2$ case reduces to the case without either birth or death [20], but where IM_{υ} is replaced by the same with an additional additive constant J_1. We can then directly reach a conclusion that the case of υ equal to 1 is the marginal case between the behavior dominated by the birth-death process and migration-dominated behavior. When $\upsilon>1$, the long-time evolution behavior of the system is dominated by the migration and is the same with the case without either birth or death. When $\upsilon<1$, the first equation in Eqs. (6) reduces to

$$\frac{da}{dt}\simeq J_1(1-a)^2. \tag{10}$$

This equation directly gives the asymptotic solution $a(t)\simeq 1-(J_1t)^{-1}$. The asymptotic aggregate size distribution in the long-time limit can then be obtained as follows:

$$a_k(t)\simeq A_0J_1^{-2}t^{-2}\exp(-x),\quad x=k(J_1t)^{-1}. \tag{11}$$

Equation (11) shows that for the $\upsilon<1$ case, the aggregate size distribution approaches the conventional scaling form [18]

$$a_k(t)\simeq C_0t^{-w}\Phi[k/S(t)],\quad S(t)\propto t^z, \tag{12}$$

where C_0 is a constant and $S(t)$ is the characteristic aggregate size of the system with its growth exponent z. It is clear that the evolution behavior is dominated by the birth-death process.

When $\upsilon=1$, we can solve Eq. (6) exactly and obtain the asymptotic solution of the aggregate size distribution $a_k(t)$ as follows:

$$a_k(t) \simeq A_0(IA_0+J_1)^{-2}t^{-2}\exp(-x),$$

$$x=(IA_0+J_1)^{-1}kt^{-1}. \tag{13}$$

This shows that the evolution behavior is dominated by the birth-death process and migration equally.

IV. KINETICS OF THE SYSTEM IN THE $J_1 > J_2$ CASE

In this case, Eq. (4) directly gives the relation between $A(t)$ and $a(t)$ as follows:

$$A(t)=A_0[1-a(t)]^2 e^{(J_1-J_2)t}. \tag{14}$$

From this we derive the total number of the clusters

$$M_0=\sum_{j=1}^{\infty} a_j=\frac{A}{(1-a)}=A_0(1-a)e^{(J_1-J_2)t}, \tag{15}$$

and the total mass of the system is

$$M_1=\sum_{j=1}^{\infty} j a_j=\frac{A}{(1-a)^2}=A_0 e^{(J_1-J_2)t}. \tag{16}$$

This shows that the total mass of the system no longer remains conserved, but grows exponentially with time, which is consistent with the demographic growth [18].

Considering the general moment in the $a(t)$ equation (4),

$$M_\upsilon=\sum_{j=1}^{\infty} j^\upsilon a_j=A\sum_{j=1}^{\infty} j^\upsilon a^{j-1}$$

$$=A_0(1-a)^2 e^{(J_1-J_2)t}\sum_{j=1}^{\infty} j^\upsilon a^{j-1}. \tag{17}$$

We first consider several simple cases with integral index υ.

When $\upsilon=0$, the moment is $M_0=A_0(1-a)e^{(J_1-J_2)t}$ and Eq. (4) then becomes

$$\frac{da}{dt}=[IA_0(1-a)^2 e^{(J_1-J_2)t}+J_1-J_2 a](1-a). \tag{18}$$

Introducing a new function $\alpha(t)$ to make a transformation

$$a(t)=1-\alpha(t)e^{-(J_1-J_2)t}, \tag{19}$$

we can transform the $a(t)$ equation (18) into the $\alpha(t)$ equation as follows:

$$\frac{d\alpha}{dt}=-(IA_0\alpha+J_2)\alpha^2 e^{-(J_1-J_2)t}. \tag{20}$$

Using the initial condition $\alpha(0)=1$, we solve this equation exactly and find that the function $\alpha(t)$ satisfies the following equation:

$$\frac{1}{\alpha}+\frac{IA_0}{J_2}\ln\frac{1+\dfrac{J_2}{IA_0}}{1+\dfrac{J_2}{IA_0\alpha}}=\frac{J_1}{J_1-J_2}-\frac{J_2}{J_1-J_2}e^{-(J_1-J_2)t}. \tag{21}$$

Then the asymptotic solution of $\alpha(t)$ at large times is found to be a constant α_0 that satisfies

$$\frac{1}{\alpha_0}+\frac{IA_0}{J_2}\ln\frac{1+\dfrac{J_2}{IA_0}}{1+\dfrac{J_2}{IA_0\alpha_0}}\simeq\frac{J_1}{J_1-J_2}. \tag{22}$$

In the end, we find the asymptotic solution for $a(t)$,

$$a(t)\simeq 1-\alpha_0 e^{-(J_1-J_2)t}, \tag{23}$$

and we further obtain the asymptotic solution of $a_k(t)$ in the long-time limit as follows:

$$a_k(t)\simeq A_0\alpha_0^2 e^{-(J_1-J_2)t}\exp(-x), \quad x=\alpha_0 k e^{-(J_1-J_2)t}. \tag{24}$$

This shows that the aggregate size distribution obeys the generalized scaling behavior

$$a_k(t)\simeq C_0[f(t)]^{-w}\Phi[k/S(t)], \quad S(t)\propto[f(t)]^z, \tag{25}$$

with $f(t)=e^t$ and the exponents are $w=z=J_1-J_2$. In this case, the total number of the aggregates $M_0=A_0(1-a)e^{(J_1-J_2)t}\simeq\alpha_0 A_0$ keeps a constant in the long-time limit.

When $\upsilon=1$, the moment is $M_1=A_0 e^{(J_1-J_2)t}$ and Eq. (4) then becomes

$$\frac{da}{dt}=[IA_0(1-a)e^{(J_1-J_2)t}+J_1-J_2 a](1-a). \tag{26}$$

This equation can be solved exactly to yield

$$a(t)=1-\frac{e^{-(J_1-J_2)t}}{IA_0 t-\dfrac{J_2}{J_1-J_2}e^{-(J_1-J_2)t}+\dfrac{J_1}{J_1-J_2}}, \tag{27}$$

and in the long-time limit, it becomes

$$a(t)\simeq 1-(IA_0 t)^{-1}e^{-(J_1-J_2)t}. \tag{28}$$

The asymptotic solution of $a_k(t)$ can then be obtained,

$$a_k(t)\simeq A_0(IA_0 t)^{-2}e^{-(J_1-J_2)t}\exp(-x),$$

$$x=k(IA_0 t)^{-1}e^{-(J_1-J_2)t}. \tag{29}$$

It obeys a further generalized scaling behavior

$$a_k(t) \simeq C_0[f(t)]^{-w_1}[g(t)]^{-w_2}\Phi[k/S(t)],$$

$$S(t) \propto [f(t)]^{z_1}[g(t)]^{z_2}, \quad f'(t), \quad g'(t)>0, \tag{30}$$

where $f(t)$ and $g(t)$ are unusual functions of time, such as e^t, ln$t$, 2^t, and so on. Here $f(t)=t$ and $g(t)=e^t$. In this case, the total number of the aggregates, $M_0 = A_0(1-a)e^{(J_1-J_2)t} \simeq (It)^{-1}$, decays with time as t^{-1} in the long-time limit.

When $\upsilon=2$, the moment is

$$M_2 = \Sigma_{k=1}^{\infty} k^2 a_k = A_0(1+a)(1-a)^{-1}e^{(J_1-J_2)t}$$

and Eq. (4) then becomes

$$\frac{da}{dt} = [IA_0(1+a)e^{(J_1-J_2)t} + J_1 - J_2 a](1-a). \tag{31}$$

In the long-time limit, it is obvious that $IA_0(1+a)e^{(J_1-J_2)t} \gg J_1 - J_2 a$. Equation (31) can be reduced to

$$\frac{da}{dt} \simeq IA_0(1+a)(1-a)e^{(J_1-J_2)t}. \tag{32}$$

We solve this equation to get the solution for $a(t)$,

$$a(t) \simeq 1 - 2e^{-\gamma_1 e^{(J_1-J_2)t}}, \tag{33}$$

where $\gamma_1 = 2IA_0/(J_1-J_2)$, and we further obtain the long-time asymptotic solution of $a_k(t)$ as

$$a_k(t) \simeq 4A_0 e^{-2\gamma_1 e^{(J_1-J_2)t}} e^{(J_1-J_2)t}\exp(-x),$$

$$x = 2ke^{-\gamma_1 e^{(J_1-J_2)t}}. \tag{34}$$

This shows that the aggregate size distribution obeys a very complicated scaling behavior

$$a_k(t) \simeq C_0\{f([g(t)]^{w_2})\}^{-w_1}[g(t)]^{w_3}\Phi[k/S(t)],$$

$$S(t) \propto \{f([g(t)]^{z_2})\}^{z_1}[g(t)]^{z_3}, \quad f'(t), g'(t)>0, \tag{35}$$

where $f(t)$ and $g(t)$ are unusual functions of time, such as e^t, ln$t$, 2^t, and so on. Here $f(t)=e^t$ and $g(t)=e^t$. In this case, the total number of aggregates is

$$M_0 \simeq 2A_0 e^{-\gamma_1 e^{(J_1-J_2)t}} e^{(J_1-J_2)t}. \tag{36}$$

It decays much faster than that in the $\upsilon=1$ case.

Now we turn to the general cases. From Eqs. (14)–(16), we have

$$a = 1 - \frac{M_0}{M_1}, \quad A = A_0(1-a)^2 e^{(J_1-J_2)t} = \frac{M_0^2}{M_1}. \tag{37}$$

The average cluster size becomes $S(t)=M_1/M_0 \gg 1$ in the long-time limit because of the aggregation effect. The cluster distribution $a_k(t)$ can be expressed in the uniform scaling form as

$$a_k(t) \simeq \frac{[M_0(t)]^2}{M_1(t)}\exp(-x), \quad x = k/S(t). \tag{38}$$

Now we investigate the kinetic behavior of $a_k(t)$ through finding the asymptotic solution of $M_0(t)$. Summing up the governing rate equation (2), we obtain the equation about the zero-order moment M_0,

$$\frac{dM_0}{dt} = -(IM_\upsilon + J_2)a_1, \tag{39}$$

where $a_1 = A = M_0^2/M_1$ and the initial condition is $M_0(0) = A_0$. Here we also try to find out the asymptotic solution of $M_0(t)$ at large times. In the long-time limit, using the scaling form of $a_k(t)$ of Eq. (38) we obtain the estimation of $M_\upsilon(t)$ as

$$M_\upsilon(t) = \sum_{j=1}^{\infty} j^\upsilon a_j \simeq \left[\frac{M_1(t)}{M_0(t)}\right]^{\upsilon+1}\frac{[M_0(t)]^2}{M_1(t)}\int_0^\infty x^\upsilon e^{-x}dx$$

$$= \Gamma(1+\upsilon)[M_1(t)]^\upsilon[M_0(t)]^{1-\upsilon}. \tag{40}$$

The M_0 equation (39) can further be written approximately as follows:

$$\frac{dM_0}{dt} \simeq -[I\Gamma(1+\upsilon)M_1^\upsilon M_0^{1-\upsilon} + J_2]\frac{M_0^2}{M_1}. \tag{41}$$

Because $M_0/M_1 \ll 1$ in the long-time limit, we can solve the above equation and obtain the asymptotic solution of M_0 in several cases.

When $\upsilon<0$, Eq. (41) reduces to

$$\frac{dM_0}{dt} \simeq -J_2\frac{M_0^2}{M_1}. \tag{42}$$

This equation directly gives the asymptotic solution $M_0 \simeq C_1 A_0$. Here the constant C_1 cannot be determined because the initial condition is not valid for the asymptotic M_0 equation. The aggregate size distribution $a_k(t)$ in the long-time limit can then be obtained from Eq. (38),

$$a_k(t) \simeq C_1^2 A_0 e^{-(J_1-J_2)t}\exp(-x), \quad x = C_1 k e^{-(J_1-J_2)t}, \tag{43}$$

which is the same as the solution in the $\upsilon=0$ case obeying the generalized scaling behavior as Eq. (25) with $f(t)=e^t$ and the exponents $w=z=J_1-J_2$. Moreover, using the moment $M_{-1} = \Sigma_{j=1}^{\infty} j^{-1}a_j = -A_0(1-a)^2\{[\ln(1-a)]/a\}e^{(J_1-J_2)t}$, we can derive the $a(t)$ equation for this case from Eq. (4) as follows:

$$\frac{da}{dt} = \left[-IA_0(1-a)^3\frac{\ln(1-a)}{a}e^{(J_1-J_2)t} + J_1 - J_2 a\right](1-a). \tag{44}$$

TABLE I. Summary of the results in $J_1 > J_2$ cases.

Case	Asymptotic behavior of the aggregate size distribution $a_k(t)$
$\upsilon < 1$	Obeys the generalized scaling law dominated by the birth-death process $a_k(t) \simeq C^2 A_0 e^{-(J_1-J_2)t} \exp(-x)$, $x = Cke^{-(J_1-J_2)t}$
$\upsilon = 1$	Obeys the further generalized scaling law dominated by the birth-death process $a_k(t) \simeq A_0 (IA_0 t)^{-2} e^{-(J_1-J_2)t} \exp(-x)$, $x = k(IA_0 t)^{-1} e^{-(J_1-J_2)t}$
$1 < \upsilon < 2$	Obeys the generalized scaling law dominated by both migration and birth-death processes $a_k(t) \simeq C_4^2 A_0 e^{-[\upsilon/(2-\upsilon)](J_1-J_2)t} \exp(-x)$, $x = C_4 k e^{-[1/(2-\upsilon)](J_1-J_2)t}$
$\upsilon = 2$	Obeys the much complicated scaling law dominated by both migration and birth-death processes $a_k(t) \simeq 4A_0 e^{-[4IA_0/(J_1-J_2)]\ e^{(J_1-J_2)t}} \times e^{(J_1-J_2)t} \exp(-x)$, $x = 2ke^{-[2IA_0/(J_1-J_2)] e^{(J_1-J_2)t}}$

From this equation, we also obtain the same results as in the $\upsilon = 0$ case of Eqs. (23) and (24) with $\alpha_0 = J_1/J_2 - 1$.

When $\upsilon = 1$, we find $I\Gamma(1+\upsilon) M_1^{\upsilon} M_0^{1-\upsilon} = IM_1 \gg J_2$ and Eq. (41) reduces to

$$\frac{dM_0}{dt} \simeq -IM_0^2. \tag{45}$$

It directly gives $M_0 \simeq (It)^{-1}$ and the same result $a_k(t)$ of Eq. (29), which is obtained from the $a(t)$ equation.

Now we turn to the general $\upsilon > 0$ case. From the above results of $M_0 \simeq \alpha_0 A_0$ in the $\upsilon = 0$ case and $M_0 \simeq (It)^{-1}$ in the $\upsilon = 1$ case, we can find that M_1^{υ} grows with time much faster than $M_0^{1-\upsilon}$ decreases in the $0 < \upsilon < 1$ case. So for the general $\upsilon > 0$ case, we can draw a conclusion that $I\Gamma(1+\upsilon) M_1^{\upsilon} M_0^{1-\upsilon} \gg J_2$ in Eq. (41) , and we finally derive the M_0 equation for the general case of $\upsilon > 0$ as

$$\frac{dM_0}{dt} \simeq -I\Gamma(1+\upsilon) M_0^{3-\upsilon} M_1^{\upsilon-1}. \tag{46}$$

When $0 < \upsilon < 1$, we find the solution of M_0 as

$$M_0 \simeq C_2 A_0, \tag{47}$$

where the constant C_2 cannot be determined because the initial condition is not valid. The aggregate size distribution $a_k(t)$ can then be obtained from Eq. (38) as follows:

$$a_k(t) \simeq C_2^2 A_0 e^{-(J_1-J_2)t} \exp(-x), \quad x = C_2 k e^{-(J_1-J_2)t}. \tag{48}$$

Meanwhile, for the $1 < \upsilon < 2$ case we find the solution of Eq. (46) as

$$M_0 \simeq C_3 A_0 e^{-\gamma_2 (J_1-J_2)t}, \tag{49}$$

where $\gamma_2 = (\upsilon - 1)/(2-\upsilon)$ and $C_3 = [\gamma_2 (J_1 - J_2)/\Gamma(1+\upsilon) IA_0]^{1/(2-\upsilon)}$. The aggregate size distribution $a_k(t)$ can then be obtained from Eq. (38) as follows:

$$a_k(t) \simeq C_3^2 A_0 e^{-\upsilon(J_1-J_2)t/(2-\upsilon)} \exp(-x),$$

$$x = C_3 k e^{-(J_1-J_2)t/(2-\upsilon)}. \tag{50}$$

This shows $a_k(t)$ satisfies the generalized scaling behavior as Eq. (25) with the scaling function $f(t) = e^t$. The scaling exponents are $w = \upsilon(J_1 - J_2)/(2-\upsilon)$ and $z = (J_1 - J_2)/(2-\upsilon)$. However, the $M_0(t)$ equation (46) doesnot give a real solution in the case of $\upsilon > 2$. It implies that the system may also undergo a gelationlike transition in this case and there is no longer the scaling behavior.

V. SUMMARY

In summary, we have proposed an irreversible aggregation model driven by migration and birth-death processes with the symmetric migration rate kernel $K(k;j) = K'(k;j) = Ikj^{\upsilon}$, and the birth rate $J_1 k$ and death rate $J_2 k$ proportional to the aggregate's size k. Based on the mean-field theory, we investigated the evolution behavior of the aggregate size distribution through developing the scaling theory. We found that the large-time behavior of the aggregate size distribution depends on the competition among three reaction rates. When the rate kernel of birth J_1 and that of death J_2 are equal, the total mass M_1 is reserved and the aggregate size distribution $a_k(t)$ obeys the conventional or generalized scaling law for the case of migration rate kernel index $\upsilon \leq 2$. The migration will play a more and more important role on the kinetic behavior of the system as the index υ increases.

When $J_1 > J_2$, the total mass M_1 is no longer reserved and increases exponentially with time as the demographic growth reveals. In this case, the aggregate size distribution $a_k(t)$ obeys the generalized or much complicated scaling law in the $\upsilon \leq 2$ case as illustrated in Table I and the migration will play a more and more important role as υ increases.

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